Unit 10 – Motion and Rates

Tuesday 2/11 | **Today's Topic:** Position, Velocity, and Acceleration

In-Class Examples: Notes Handout

AP Multiple Choice

A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t+2)$. What is the acceleration of the particle at time $t = 6$?

(A) 1.500 (B) 20.453 (C) 29.453 (D) 74.860 (E) 133.417

A particle moves along the x-axis with its position at time t given by $x(t) = (t-a)(t-b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?

(A) $t = ab$

(B) $t = \dfrac{a+b}{2}$

(C) $t = a + b$

(D) $t = 2(a+b)$

(E) $t = a$ and $t = b$

Homework: Worksheet 84

Wednesday 2/12 | **Today's Topic:** Position, Velocity, and Acceleration

In-Class Examples: Notes Handout

AP Multiple Choice

For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

A particle moves on the x-axis so that at any time t, $0 \leq t \leq 1$, its position is given by $x(t) = \sin(2\pi t) + 2\pi t$. For what value of t is the particle at rest?

(A) 0 (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{1}{2}$ (E) 1

Homework: Worksheet 85

Thursday 2/13 | **Today's Topic:** Motion and Graphs of Velocity

In-Class Examples: Interpreting the Graph of Velocity Handout

AP Multiple Choice

A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at time $t = 3$?

(A) 13 (B) 15 (C) 16 (D) 17 (E) 25

A particle moves along a straight line so that at time $t > 0$ the position of the particle is given by $s(t)$, the velocity is given by $v(t)$, and the acceleration is given by $a(t)$. Which of the following expressions gives the average velocity of the particle on the interval $[2, 8]$?

(A) $\frac{1}{6}\int_2^8 a(t)\,dt$

(B) $\frac{1}{6}\int_2^8 s(t)\,dt$

(C) $\frac{s(8) - s(2)}{6}$

(D) $\frac{v(8) - v(2)}{6}$

(E) $v(8) - v(2)$

Homework: Worksheet 86

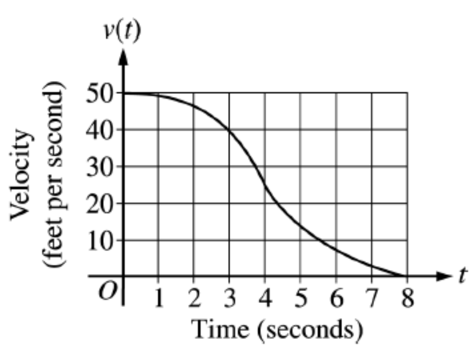
Friday 2/14 | **Today's Topic:** Motion – Tables of Values (Velocity)

In-Class Examples: Interpreting Tables of Values (Velocity)

AP Multiple Choice

A particle moves along a straight line with velocity given by $v(t) = 5 + e^{t/3}$ for time $t \geq 0$. What is the acceleration of the particle at time $t = 4$?

(A) 0.422 (B) 0.698 (C) 1.265 (D) 8.794 (E) 28.381



The graph above gives the velocity, v, in ft/sec, of a car for $0 \leq t \leq 8$, where t is the time in seconds. Of the following, which is the best estimate of the distance traveled by the car from $t = 0$ until the car comes to a complete stop?

(A) 21 ft (B) 26 ft (C) 180 ft (D) 210 ft (E) 260 ft

Homework: Worksheet 87

Tuesday 2/18	**Today's Topic:** Motion Review

In-Class Examples: None

AP Multiple Choice

A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

(A) -10 (B) -5 (C) -3 (D) 3 (E) 17

An object moves along a straight line so that at any time $t \geq 0$ its velocity is given by $v(t) = 2\cos(3t)$. What is the distance traveled by the object from $t = 0$ to the first time that it stops?

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{2}{3}$ (D) $\frac{\pi}{3}$ (E) $\frac{4}{3}$

Homework: Worksheet 88

Wednesday 2/19	**Today's Topic:** Motion Quiz (Reading a graph of v(t))

In-Class Examples: None

Homework: None

Thursday 2/20	**Today's Topic:** Integral as Net Change

In-Class Examples:

Ex. 1 The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \leq t \leq 4$ hours. How much sewage has entered the tank during the time interval?

Ex. 2 The tide removes sand from Sandy Point Beach at a rate modeled by the function R given by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$ cubic yards per hour. How much sand will have been removed by the tide after 6 hours?

Ex. 3 A spherical tank contains 81.637 gallons of water at time $t = 0$ minutes. For the next 6 minutes, water flows out of the tank at a rate of $9\sin\left(\sqrt{t+1}\right)$ gallons per minute. How many gallons of water are in the tank at the end of 6 minutes?

AP Multiple Choice

A home uses fuel oil at the rate $r(t) = 10 + 8\sin\left(\frac{t}{60}\right)$ gallons per day, where t is the number of days from the beginning of the heating season. To the nearest gallon, what is the total amount of fuel oil used from $t = 0$ to $t = 60$ days?

(A) 7 gal (B) 14 gal (C) 600 gal (D) 821 gal (E) 1004 gal

At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628

Homework: Worksheet 89

Monday 2/24	**Today's Topic:** Integral as Net Change

In-Class Examples:

Ex. 1
A population of insects increases at a rate of $R(t) = 200 + 10t + 3t^2$ insects per day. There are 35 insects present when t =2 days.

(a) Write a function $P(t)$ that can be used to determine the number of insects present after t days.
(b) How many insects are present after 7 days?

Ex. 2 A survey shows that a presidential candidate is gaining supporters at a rate of $R(t) = 2t + 25$ people per day, where t is the number of days since he announced his candidacy. He initially has 500 supporters.

(a) Write a function $S(t)$ that can be used to determine the number of supporters the candidate has after t days.
(b) How many supporters will the candidate have after 60 days?

AP Multiple Choice

For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

(A) The change in temperature during the first day
(B) The change in temperature during the 24th hour
(C) The average rate at which the temperature changed during the 24th hour
(D) The rate at which the temperature is changing during the first day
(E) The rate at which the temperature is changing at the end of the 24th hour

Homework: Worksheet 90

Tuesday 2/25	**Topic:** Rates and Motion FRQ

In-Class Examples: AP Multiple Choice

AP Multiple Choice

The height above the ground of a passenger on a Ferris wheel t minutes after the ride begins is modeled by the differentiable function H, where $H(t)$ is measured in meters. Which of the following is an interpretation of the statement $H'(7.5) = 15.708$?

(A) The Ferris wheel is turning at a rate of 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(B) The Ferris wheel is turning at a rate of 15.708 meters per minute 7.5 minutes after the ride begins.

(C) The passenger's height above the ground is increasing by 15.708 meters per minute when the passenger is 7.5 meters above the ground.

(D) The passenger's height above the ground is increasing by 15.708 meters per minute 7.5 minutes after the ride begins.

(E) The passenger is 15.708 meters above the ground 7.5 minutes after the ride begins.

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t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

A function $f(t)$ gives the rate of evaporation of water, in liters per hour, from a pond, where t is measured in hours since 12 noon. Which of the following gives the meaning of $\int_4^{10} f(t)\,dt$ in the context described?

(A) The total volume of water, in liters, that evaporated from the pond during the first 10 hours after 12 noon

(B) The total volume of water, in liters, that evaporated from the pond between 4 P.M. and 10 P.M.

(C) The net change in the rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(D) The average rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(E) The average rate of change in the rate of evaporation, in liters per hour per hour, from the pond between 4 P.M. and 10 P.M.

Homework: Worksheet 91

Wednesday 2/26	Today's Topic: Rates and Motion Review

In-Class Examples:

A graphing calculator is required for some problems or parts of problems.

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

Homework: Worksheet 92

Thursday 2/27	Today's Topic: Rates and Motion Review

In-Class Examples: None

Homework: Worksheet 93

Friday 2/28	Today's Topic: Rates and Motion Exam

In-Class Examples: None

Homework: None